Mechanism for granular segregation

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A process is described that produces horizontal size segregation in a vertically vibrated layer of granular material. The behavior is a consequence of two distinct phenomena that are unique to excited granular media: vibration, which causes the large particles to rise to the top of the layer, and a vibrating base with a sawtooth surface profile, which can produce stratified flows in opposite directions at different heights within the layer. The result of combining these effects is that large and small particles are horizontally driven in opposite directions. The observations reported here are based on computer simulations of granular models in two and three dimensions.

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Granular media have been said to embody properties of solids, liquids, and gases, but in actual fact studies of the granular state continue to produce surprising results that are unique to this class of matter [1–3]. Computer modeling plays an important role in these developments, due to the fact that once it has demonstrated an ability to reproduce the observed behavior it can be used in trying to understand the mechanisms involved; examples of the dynamics of granular media that have been examined by this kind of approach include vertical size segregation by both vibration [4,5] and shear [6,7], and vibration-induced surface waves [8–11].

A further instance of unusual behavior in granular matter that has come to light very recently involves a granular layer that is vibrated vertically by a base whose surface profile consists of sawtoothlike grooves (in the experimental realization the material is contained in a narrow annular region between two upright cylinders and is thus essentially two dimensional). What is observed [12], both experimentally and in computer simulation, is the occurrence of horizontal flow. Moreover, the flow direction and magnitude depend on many of the parameters required to specify the system in an apparently complex manner. Subsequent more detailed simulation of this phenomenon [13] revealed that the induced flow rate actually varies with height within the granular layer; even more surprising is the observation that oppositely directed flows can exist simultaneously at different levels in the layer. Once the strongly stratified nature of the horizontal flow is appreciated, the sensitive parameter dependence of the overall horizontal motion of the layer is readily understood as being a consequence of the competition between opposing stratified flows.

A combination of two of the phenomena that are unique to granular matter, namely, the already familiar vertical size segregation under vibration, the so-called "Brazil nut" effect, and the recently observed stratified flow outlined above, suggests an entirely different mechanism for separating the components of a granular mixture according to particle size. If, under suitable vibration conditions, the upper and lower layers of the material are able to move horizontally in opposite directions under the influence of a sawtooth-shaped base, and the same vibrations cause the large particles to climb toward the top of the layer, then it is apparent that some degree of horizontal segregation of large and small particles should occur as a consequence. A process based on behavior of this kind could well be significant from an industrial perspective. In order to determine whether this proposed segregation actually occurs, computer simulations based on a discrete-particle model for granular material have been carried out in both two and three dimensions. Most of the results are for the former, which is essentially equivalent to a very narrow upright container, since considerably smaller numbers of particles are involved in the calculations.

The granular model employed here is the same as in the horizontal flow simulations [13]; early work with simplified, inelastic, soft-particle models of this kind appeared in [14,6], and a number of similar models are now in widespread use for modeling the dynamics of granular flow. The interaction used to provide the spherical shape of the granular particles has a Lennard-Jones (LJ) form, truncated at a range r_c where the repulsive force falls to zero. For particles located at $\boldsymbol{r}_i$ and $\boldsymbol{r}_j$ this is $\boldsymbol{f}_{ij}^r = (48\epsilon/r_{ij})[(\sigma_{ij}/r_{ij})^{12} - 0.5(\sigma_{ij}/r_{ij})^6]\hat{\boldsymbol{r}}_{ij}$ for $r_{ij} < r_c = 2^{1/6}\sigma_{ij}$; here $\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j$ and $\sigma_{ij} = (\sigma_i + \sigma_j)/2$. σ_i is the approximate diameter of particle i, although because the particles are slightly soft the diameter is not precisely defined. Normal and transverse damping forces (the latter is optional) act during the collision (i.e., while $r_{ij} < r_c$); the normal damping force is $\boldsymbol{f}_{ij}^n = -\gamma_n(\dot{\boldsymbol{r}}_{ij} \cdot \hat{\boldsymbol{r}}_{ij})\hat{\boldsymbol{r}}_{ij}$, and the transverse force is $\boldsymbol{f}_{ij}^s = -\min(\mu|\boldsymbol{f}_{ij}^r + \boldsymbol{f}_{ij}^n|, \gamma_s|\boldsymbol{v}_{ij}^s|)\hat{\boldsymbol{v}}_{ij}^s$, where $\boldsymbol{v}_{ij}^s$ is the relative transverse velocity at the point of contact (which depends on the angular velocities of the particles and their relative translational velocity). The value of the static friction coefficient is $\mu = 0.5$, and the normal and transverse damping coefficients are $\gamma_n = \gamma_s = 5$. Particles are also subject to a gravitational acceleration g.

For convenience the results are expressed in terms of reduced units, in which length measurements are based on the diameter of the small particles. Such units are readily converted to actual physical units. If, for example, the diameter is actually 10^{-3} m, then the reduced time unit consistent with the value $g = 5$ used subsequently corresponds to 0.022 s, so that the vibration frequency value $f = 0.4$ employed in these simulations is equivalent to ≈ 18 Hz, typical of the

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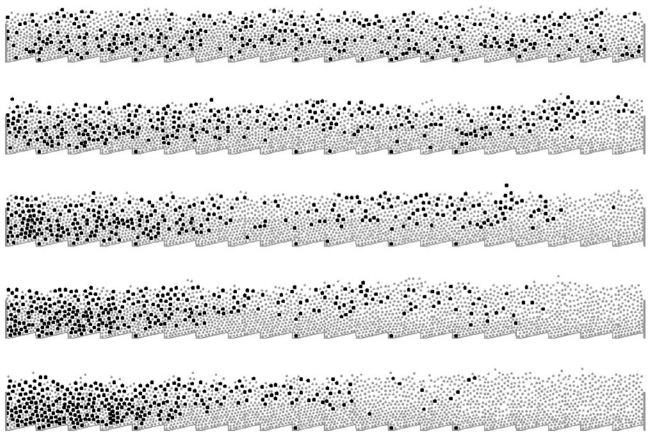

FIG. 1. Screen images showing stages in the size segregation process for a two-dimensional system; the large particles are darkly shaded.

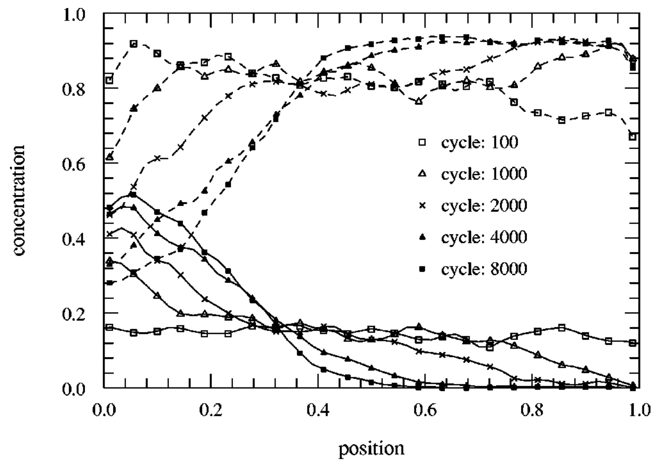

FIG. 2. Variation of the local concentration of large and small particles (solid and dashed curves, respectively) across the system at several stages during the segregation process.

values used experimentally. Horizontal flow rates [13] within the strata are then of order 10^{-2} m/s.

The particle size distribution is bimodal, with values randomly distributed over narrow ranges extending from the nominal small and large sizes to values 10% lower. All particles have the same material density. The sawtooth base for the two-dimensional simulations is constructed (for reasons of computational convenience) from a set of grainlike particles positioned in a manner that describes the desired asymmetric sawtooth profile; the base particles are forced to oscillate vertically in unison to produce the effect of a sinusoidally vibrated base, and it is these vibrations that drive the system. The base particles interact with the freely moving granular particles using the same force laws as above; in order to ensure reasonably straight sawtooth edges the diameter of these particles is 0.33 and the distance between their centers is 0.17. Further discussion of the model appears in [13], and other more general aspects of the computational methodology are to be found in [15]. Unlike the earlier work, which dealt with a horizontally periodic system, particles here are confined to a container with reflecting sidewalls; the container is sufficiently high that particles never reach the upper boundary.

Figure 1 shows a series of screen snapshots (the image resolution is limited) for a mixture in which there is a 15% concentration of large particles with diameter 1.4. The first image corresponds to an early state of the system, with large particles randomly located throughout the layer. Subsequent images show the particle distributions after 1000, 2000, 4000, and 16 000 vibration cycles; the occurrence of size segregation, in this instance marked by the leftward motion of the large particles, is clearly visible.

The presence of the sidewalls interferes with the stratified horizontal flows observed with periodic boundaries (corresponding to an effectively infinite system) [13]; nevertheless, in a system of the width considered here, the stratified flow not too close to the walls is sufficiently strong to drive the segregation. Any practical implementation of such a separation mechanism would of course require some means of continuously injecting a granular mixture at a point above the layer somewhere near its center, and extracting the (at least partially) segregated products from opposite ends of the container through suitably located openings.

Concentration profiles for the large and small particles at various times are shown in Fig. 2. The gradual emergence of segregation is apparent, beginning with an initially uniform system, and finishing with almost all the large particles having migrated to the left side of the container. The system width is 180 (in reduced units) for the particular case shown here, and the nominal number of layers is 8. The frequency is $f=0.4$ and the amplitude $A=1$. The dimensionless acceleration $\Gamma=(2\pi f)^2 A/g$ is an important quantity in vibrating systems, with $\Gamma\approx 1$ the minimum required to excite the layer; a value $\Gamma=1.26$ is used here, obtained by setting $g=5$. The base contains 20 sawteeth of height 2, with an asymmetry such that the right edge of each tooth is essentially vertical. For the case of granular particles with a unimodal size distribution [13] this choice of parameters produced oppositely directed stratified flows of similar magnitude (there the system width was just 90, so the relevant velocity profile is for ten sawteeth). The large particle size is 1.2 and the concentration 15%. Results are averaged over short time intervals and over six independent runs (with different initial states), and are smoothed to reduce fluctuations. Similar behavior is observed if the LJ overlap potential used here is replaced by linear or Hertzian potentials; thus the results do not depend on the choice of interaction. The fact that segregation occurs for a large particle size of only 1.2 suggests a highly sensitive mechanism.

In the case of the unimodal size distribution, the stratified flows were observed [13] to depend, often in a complex manner, on the various parameters used to specify the system. A similar dependence also occurs for mixtures. Space does not permit a detailed analysis of the dynamical phase diagram, which would show how the separation rate and direction (as well as the purity of the segregation products) depend on each of these parameters. Instead, a few selected examples illustrating the variation with certain key parameters will be presented, with emphasis on cases where segregation is strong.

Particularly important from a practical perspective is the

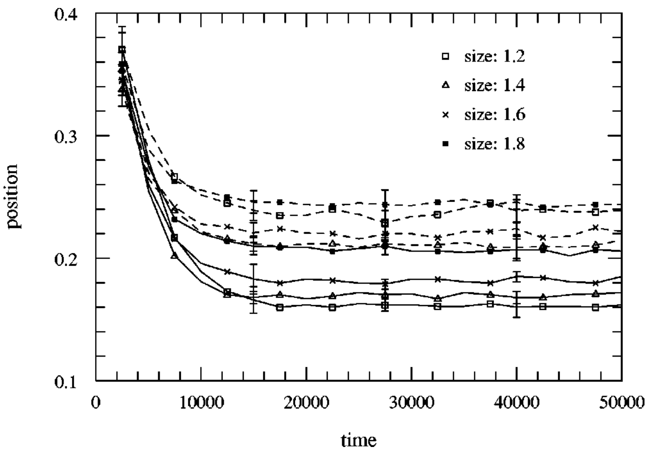

FIG. 3. Position of the center of mass of the large particles as a function of time for different particle sizes, and for 20 and 40 sawteeth (solid and dashed curves).

efficiency of the segregation mechanism for different large particle sizes. Figure 3 shows this size dependence for two sawtooth widths. The degree of segregation is expressed in terms of the time-dependent position of the center of mass of the large particles; the occasional particle trapped in a sawtooth groove before its migration is complete will adversely affect such a measurement, as will any other effects preventing the formation of a well-defined vertical boundary between the two segregated species (see below). The results are again averaged over six independent runs. The final value depends on the large particle size; in the case of 20 sawteeth it increases monotonically with increasing size, reflecting the increasing space occupied by the large particles, but for 40 sawteeth the results for diameter 1.2 deviate from the expected sequence. The times required to reach the limiting position are essentially the same.

While a detailed picture of the particle movement over the entire history of the segregation is not readily obtained because of the strong degree of mixing that occurs, a study of the trajectories for a subset of particles, once the process has effectively ended, demonstrates where the particles tend to be located. Figure 4 shows short trajectory segments for just 2% of the particles after 35 000 vibration cycles; each trajectory consists of five points at 100-cycle intervals. In the first two examples (with 20 relatively wide sawteeth, and large particle sizes 1.4 and 1.2) the large particles congregate on the left; they occupy a region that includes the section of the layer abutting the wall extending from top to bottom, and adjacent to this a roughly wedge-shaped region that tapers upward to the right. The nonrectangular shape of this region explains why the center of mass position does not fully characterize the final state. In the third example (80 very narrow sawteeth, particle size 1.2) segregation occurs in the opposite direction and is less complete.

In those cases where large particles migrate to the left, the shape is a consequence of a dynamic equilibrium involving both stratified flow, in which particles in the upper levels are driven horizontally to the left by the sawteeth and lower particles to the right, and Brazil-nut ‘‘buoyancy’’ which raises the large particles. In a practical implementation of such a segregation apparatus, large particles would be removed near the upper left corner and small particles via an opening near the lower right. The case where large particles migrate to the right reflects the absence of leftward stratified flow due to the sawtooth base. In all cases shown here, irrespective of direction, the horizontally driven large particles eventually displace the small particles from the appropriate sidewall.

The corresponding three-dimensional version of this system consists of a rectangular base with linear sawtoothlike grooves; similar results would be expected from simulations of such a system. Since it is more interesting to consider a problem involving an alternative geometry, a system with circular symmetry is described here.

In this three-dimensional example the base consists of a set of concentric circular grooves, shaped to produce a radial profile identical to that used in two dimensions. Interactions between particles and the base are handled with a simple extension of the approach used above, by employing a series of concentric donutlike base ‘‘particles’’ to represent the grooves (only the radial coordinates of such particles enter into the force computation). Since a substantially larger number of granular particles are necessary to achieve a linear size similar to that of the two-dimensional system, only a limited exploration of this problem has been undertaken. The transverse damping force is omitted here; while its absence does not seriously influence the observed behavior, indicating once again that the phenomenon is not especially sensitive to the details of the model, it does reduce the computational effort.

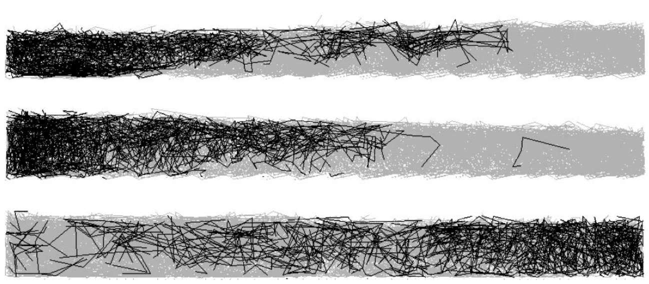

FIG. 4. Trajectories of 2% of the particles after segregation has ended (large particle trajectories are in black), for pairs of values of large particle size and number of sawteeth (1.4, 20), (1.2, 20), and (1.2, 80).

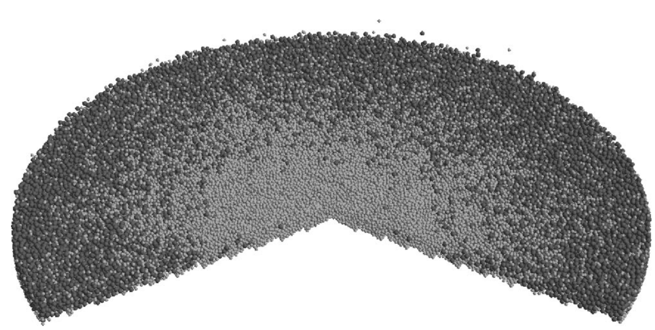

FIG. 5. Segregation in a three-dimensional system; a wedge-shaped region has been removed from the front and the container is not shown (large particles are darker).

Figure 5 is a screen image from one such simulation showing the system after 2000 vibration cycles. The container diameter used here is 180 and the nominal layer thickness is 8; there are 11 concentric sawtooth grooves, and the large particles have size 1.5 and concentration 33%. The sawtooth profile, whose influence is apparent in the positioning of the particles closest to the base, is oriented so that large particles should migrate toward the outer boundary of the cylindrical container while small particles move inward, which indeed is exactly what is seen to occur.

In summary, computer simulations employing a type of granular model whose viability has been established in other studies of grain flow have been used to investigate a mechanism for achieving size segregation. Selected results have been presented here, with a more detailed determination of the parameter dependence to be published in due course. Since separation and sorting are essential functions in the processing of bulk granular materials, such an approach, implemented, for example, as a continuous flow device, has potential industrial value. It remains to be seen whether the behavior of real granular matter is in accord with the predictions of these simulations.

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